

GLUON CONDENSATE AND BEYOND

The 1999 Sakurai Prize Lecture ^a

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We review briefly and in retrospect the development which brought about the QCD sum rules based on introduction of the gluon condensate (M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov (1978)).

Introduction. QCD '76-'77

This review is based on the Sakurai prize lecture given at the Centennial Meeting of the American Physical Society (Atlanta, March 1999). The lecture was the second in the series of three talks related to the Sakurai prize 1999, and it followed Arkady Vainshtein's summary of the discovery of the penguins ¹. My main task was to review the QCD sum rules ² within the context of the time when they were uncovered, that is the years 1976-79. Roughly speaking, the QCD sum rules relate properties of resonances, such as mass and leptonic width of, say, ρ -meson to the vacuum properties which are parameterized in terms of quark and gluon condensates.

By the year 1976, QCD was of course highly appreciated and quite far developed. One may say that the basic ideas constituting the present-day understanding of QCD were already put on the table although in somewhat less coherent way than we know them today. In particular, I would emphasize three points related, to different extent, to what we are going to discuss later:

(i) Asymptotic freedom ³ was famous by that time. The effective coupling tends to zero at short distances, or high momenta Q :

$$\alpha_s(Q^2) \approx \frac{1}{b_0 \ln(Q^2/\Lambda_{QCD}^2)}. \quad (1)$$

Moreover, since up- and down-quarks are practically massless the only dimensional parameter in QCD is the ultraviolet cut off Λ_{UV} . Hence, it was known very well that the observable hadronic masses should be generated via the so called dimensional transmutation:

$$m_N \approx (const)\Lambda_{UV} \exp(-b_0/2\alpha_s(\Lambda_{UV}^2)). \quad (2)$$

Although the relation looks rather exotic, the QCD sum rules, as we shall see, make a half way to realize it.

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(ii) It was known that the perturbative expansion by itself, even taken to an infinite order, cannot explain why the η' -meson is heavy. Moreover, an example of a non-perturbative vacuum field, that is the instanton solution⁴ has been found. Generically, instantons do solve the η' -problem⁵. Also, instability of the QCD vacuum with respect to formation of nonperturbative color magnetic field had been conjectured⁶.

(iii) The dual-superconductor model of the quark confinement had been proposed⁷. The basic idea behind the model is that the properties of the QCD vacuum are similar to the properties of ordinary superconductor. Indeed, if a pair of magnetic charges is introduced into a superconductor the potential energy of the pair would grow linearly with the distance r at large distances:

$$\lim_{r \rightarrow \infty} V(r) \approx \sigma_{\infty} \cdot r \quad (3)$$

where σ_{∞} is the tension of the Abrikosov-Nielsen-Olesen string⁸. In QCD, a similar phenomenon was postulated to happen, with a change of magnetic charges to (color) electric, or dual charges.

Because of the asymptotic freedom (AF) the quantitative predictions of QCD referred to short distances while access to non-perturbative effects (see, e.g., points (ii), (iii) above) was blocked by infrared divergences. The basic strategy adopted in² was to exploit the power of perturbative expansions at short distances and, abandoning for the moment the ambitious program of tampering and understanding infrared sensitive contributions, simply parameterize them in terms of a very few numbers.

This mini-review is in three parts:

1. Resonance properties and asymptotic freedom.
2. Gluon condensate.
3. Further developments.

Thus, we are going to review first sum rules which constrain resonance properties basing exclusively on the AF⁹. Then we will introduce the idea² that it is vacuum condensates, which limit the validity of the AF at moderate momenta Q^2 . Finally, we will highlight a few topics related to much more recent developments during last few years. In all the cases we take the freedom of being subjective and not aiming at completeness of the presentation to any extent.

Sum Rules

It is intrinsic to the method that we are going to use that one deals not with a particular hadronic state directly but rather with sum rules. This is because the theoretical predictions refer to short distances and times where the effective coupling (1) is small. On the other hand, to perform a measurement on a state with a definite energy one needs long time, because of the uncertainty principle.

The method of sum rules is deeply rooted in quantum mechanics, and first sum rules are well known since long. The simplest one seems to be that the probability to find a system in one of the states is unity. Similarly, the completeness condition

reads as:

$$|n\rangle\langle n| = I \quad (4)$$

where I is the unit operator and $|n\rangle$ is a complete set of states. An example closer to our topic is provided by the Thomas-Reiche-Kuhn sum rules for dipole transitions in atoms. One starts with the canonical commutator

$$[r_i, p_k] = i\delta_{ik} \quad (5)$$

and averages it over a ground S-wave atomic state $|0\rangle$ with energy E_0 . Inserting a complete set of states into the (see Eq (4)) in the left hand side of (5) one immediately arrives at

$$\frac{3}{m} = \sum_n (E_n - E_0) |\langle 0 | r_i | n \rangle|^2. \quad (6)$$

The matrix elements of r_i are measurable in dipole electromagnetic transitions between the atomic states.

Note that the canonical commutator (5) is the same as for free particles. In this sense the situation resembles QCD where the quarks propagate at short distances the same as free particles. However, observing hadrons we would not find much quarks at short distances since they are predominantly at a characteristic distance of order Λ_{QCD}^{-1} . Therefore, to ensure that quarks do not fly away one has to resort to an external source of quarks such as electromagnetic current and consider unphysical kinematics with space-like total momentum of quarks q , $-q^2 \equiv Q^2 \gg \Lambda_{QCD}^2$. Then according to the uncertainty principle quarks can exist for time of order

$$\tau \sim \frac{1}{\sqrt{Q^2}} \quad (7)$$

which is small if Q^2 is large. For consistency, after such time the quarks are to be absorbed by another current, see Fig. 1.

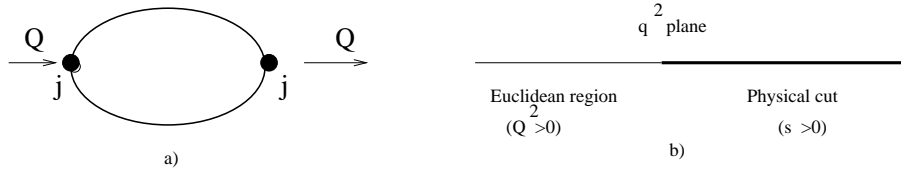


Figure 1: a) Correlator of currents in the parton model approximation. b) q^2 plane.

In the field theoretical language, we are considering in fact a correlation function $\Pi_j(Q^2)$:

$$\Pi_j(Q^2) = i \int d^4x \exp(iqx) \langle 0 | T \{ j(x), j(0) \} | 0 \rangle, \quad Q^2 \equiv -q^2 \quad (8)$$

where the current j may have various quantum number, like spin, isospin and for simplicity we do not indicate these quantum numbers, i.e. suppress the Lorenz indices and so on.

The basic theoretical ingredient is that $\Pi(Q^2)$ at large Q^2 can be calculated in the parton model approximation:

$$\lim_{Q^2 \rightarrow \infty} \Pi_j(Q^2) = \Pi_j(Q^2)_{\text{parton model}}. \quad (9)$$

On the other hand, by using dispersion relations $\Pi_j(Q^2)$ can be expressed in terms of the absorptive part which is non-vanishing only for time-like total momentum, $q^2 > 0$:

$$\Pi_j(Q^2) = \frac{1}{\pi} \int \frac{\text{Im}\Pi_j(s)}{s + Q^2} ds. \quad (10)$$

The imaginary part is directly observable, provided that the current j is a physical one. In particular, in case of the electromagnetic current, $j = j_{el}$ the imaginary part in Eq (10) is proportional to the total cross section of e^+e^- -annihilation into hadrons:

$$\text{Im}\Pi_{J_{el}}(s) = (\text{const}) \frac{(\sigma_{tot}(e^+e^- \rightarrow \text{hadrons}))}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}. \quad (11)$$

Upon substitution of (11), the Eq. (10) becomes no less a sum rule than, say, (6). Indeed, $\Pi_{j_{el}}(Q^2)$ is calculable and the same as for free particles plus small radiative corrections, while $\text{Im}\Pi(s)$ is observable.

It is worth noting that to apply the technique considered it suffices to ensure that the time which quarks exist is small indeed. Apart from imposing the condition that Q^2 is large (see Eq (7)) there exist other possibilities. In particular, as far as production of heavy quarks is concerned one can consider $Q^2 = 0$ since in that case¹⁰

$$\tau \sim 1/m_H.$$

This observation turned in fact crucial for the charmonium sum rules⁹. Also, one might consider a complex value of q^2 provided that it is still far enough from the cut $s > 0$ ¹¹.

After a second thought, one might say, however, that as far as Q^2 is large the Eq. (11) is not so much a sum rule but rather a tautology. Indeed, Feynman graphs themselves define analytical functions $\Pi_j(q^2)$. Then evaluating Π_j in deeply Euclidean region of large Q^2 is essentially the same as evaluating the corresponding cross section within the parton model directly for positive $s > 0$. For example, if we resort to Fig. 1 both to evaluate the real part of $\Pi(Q^2)$ at large Euclidean Q^2 and the imaginary part at $s > 0$ then we get a trivial relation:

$$\ln \frac{\Lambda_{UV}^2}{Q^2} = \int^{\Lambda_{UV}^2} \frac{ds}{s + Q^2}. \quad (12)$$

And, indeed, we do not seem to learn anything new beyond the famous parton model prediction that the total cross section of e^+e^- annihilation into hadrons is the same as for free quarks. This story can repeat itself order by order in $\alpha_s(Q^2)$.

Thus, for me personally the whole saga of the sum rules began not with systematic studies along the lines outlined above but rather from a conversation with Arkady Vainshtein during one of my visits to Novosibirsk. For the reasons which I do not remember at all, we discussed the positronium case. And Arkady was insisting that, on one hand, we could evaluate the polarization operator to four-loop order in the Euclidean region (see Fig. 2) while, on the other hand, in

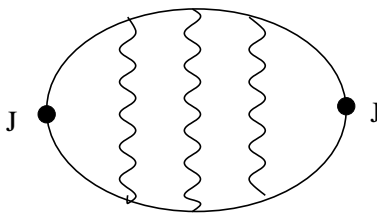


Figure 2: An example of a four-loop graph contributing to a correlator of local currents in case of QED. The wavy lines are photons and the solid lines are electrons (positrons).

the physical cross section we would have to account for the contributions of the positronium states which do not arise to any order in perturbative expansion (see Fig. 3).

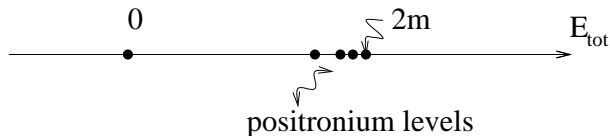


Figure 3: Structure of the q^2 plane in case of QED. Apart from the cut beginning at $2m_e$, there are positronium states below the threshold of e^+e^- production.

Indeed, the widths of the positronium states for transition induced by local currents are proportional to

$$\Gamma \sim |\Psi_{\bar{e}e}(0)|^2 \sim \alpha_{el}^3, \quad (13)$$

where $\Psi_{\bar{e}e}(0)$ is the Ψ -function at the origin, and they do contribute to the imaginary part of the polarization operator at the four-loop level.

Looking backwards, it was a proof of non-triviality of the sum rules. Still, the whole problem seemed to be of pure academic interest at best. Indeed, we did not expect at all to learn something new about QED. Moreover, it was very late at night or, better to say, early in the morning and, what was most irritating for me, it was a kind of mockery even to contemplate a possibility that I would ever be able to evaluate a four-loop graph.

Asymptotic Freedom vs Resonances

The positronium story got, however, a dramatic turn sometime later after discussions at ITEP when it was realized that the counting of the powers of α_s looks very different from the counting of the powers of α_{el} in the positronium case, see (13). The crucial point is that the effective coupling (1) changes rapidly between momenta of order Q^2 and Λ_{QCD}^2 . The parton model predictions are sensitive to $\alpha_s(Q^2)$. On the other hand, the resonances are governed by α_s of order unity at momenta of order Λ_{QCD} and not sensitive to the small $\alpha_s(Q^2)$ at all:

$$\Psi_{\bar{c}c}(0) \sim [\alpha_s(Q^2)]^0 \quad (14)$$

where $\Psi_{\bar{c}c}(0)$ is the Ψ -function at the origin of the charmonium states. Thus, even if we evaluate $\Pi_j(Q^2)$ to the lowest order in perturbation theory, in the dispersive part $Im\Pi_j(s)$ we need to keep the resonance contributions. Which means in turn that the asymptotic freedom itself constrains the resonance properties! Quantum Chromodynamics appeared to be very friendly towards people whose ability to evaluate Feynman graphs does not go beyond one loop.

We were able in fact to include two loops as well, and ended up with sum rules of the form⁹:

$$\int \frac{R_c(s) ds}{s^{n+1}} \approx \frac{A_n}{(4m_c^2)^n} (1 + B_n \alpha_s(m_c^2)), \quad (15)$$

where R_c is the contribution of the current of the charmed quarks into the ratio $R(s)$, A_n, B_n are calculable numbers, and m_c is the mass of the charmed quark (it goes without saying that the only “heavy” quark known at that time was the c -quark). The integer number n corresponds to the n -th derivative from (10) and we have chosen $Q^2 = 0$ in case of heavy quarks. Moreover, as is emphasized above we are to keep the charmonium contribution in the left-hand side of Eq. (15). Finally, the $\alpha_s(m_c^2)$ correction corresponds to the two-loop contribution, or one-gluon exchange. It was not difficult to adapt known QED results to include two loops as well. The idea was to control violations of the AF through the coefficients B_n .

The central point about the sum rules (15) is that it is not only so that we are allowed to keep the resonances but that they turn to dominate the sum rules. Indeed, a single glance at the experimental cross section of (hidden) charm production in e^+e^- annihilation reveals that the J/ψ is a huge resonance by far overshadowing the continuum production of the D -mesons. Saturating the sum rules by a single resonance we got relations like⁹

$$\Gamma_{ll}(J/\psi) \approx \frac{4\alpha_{el}^2 (A_3)^4}{27\pi (A_4)^3} M_{J/\psi} \approx 5 \text{ KeV}. \quad (16)$$

where we neglected the α_s corrections altogether. At a closer look, there were many nontrivial issues involved. Victor Novikov, Lev Okun, Misha Shifman, Arkady Vainshtein, Misha Voloshin and myself, we worked enthusiastically together and summarized the findings in an issue of the “Physics Reports”⁹.

After the initial euphoria, however, we began to evaluate the results more sober. To begin with, we were not without predecessors in relating the resonance properties to the bare quark cross section. The first in the line seemed to be J.J. Sakurai who had said that the leptonic widths of vector mesons like the ρ correspond to the quark cross section smeared over the interval of s till the next resonance, like the ρ' ¹². Indeed, our relations in case of the light quarks with isotopic spin $I = 1$ looked as:

$$\int ds \exp(-s/M^2) R^{I=1}(s) \approx \frac{3}{2} M^2 \left(1 + \frac{\alpha_s(M^2)}{\pi} \right) \quad (17)$$

where M^2 is arbitrary as far as $\alpha_s(M^2)$ is small. For M^2 about $(GeV)^2$ the numerical contribution of the ρ -meson to the left-hand side of (17) is very substantial while the right-hand side is calculable in terms of quarks. Thus, one may say that Eq (17) represents a kind of refined Sakurai duality. Refined, in the sense that the weight function with which one integrates the cross section is determined from the first principles of QCD.

The weakest point was that it was not known at which n (see Eq. (15)) or M^2 (see Eq. (17)) we should stop applying the sum rules based on the asymptotic freedom. Say, the number in the right-hand side of Eq. (16) contains a hidden dependence on the value of n for which we choose the resonance to dominate the sum rules. For further progress, we needed the mechanism of the AF breaking.

Gluon Condensate

The question “who stops the Asymptotic Freedom ?” occupied us through the whole summer of 1978 and further into the fall. At first sight, the answer was almost trivial: the growth of the coupling at lower momenta. It would be a common answer. However, gradually a feeling developed that the things are not so simple. It was difficult, however, to formalize this feeling. Still, with time some paradoxes crystallized themselves. For example, it was quite a common theoretical guess that the splitting between the vector mesons does not depend on flavor, say:

$$m(\rho') - m(\rho) \approx m(\psi') - m(J/\psi), \quad (18)$$

which is true experimentally. This simple-looking observation was, however, a serious challenge to the wisdom that it is the growth of the effective coupling (1) that stops the AF at moderate mass scales. Indeed, if expressed in terms of an invariant quantity, s , Eq. (18) implies that the J/ψ is dual to a much larger interval of s than the ρ because the c-quark is heavy. In other words, the AF is violated in the ρ channel much later than in the charmonium channel, if we start from very large Q^2 downwards. A direct numerical analysis of the sum rules confirmed this expectation. However, the coupling should run as a function of an invariant quantity and is flavor blind.

The crucial step was to explore the possibility that it is the soft nonperturbative vacuum fields that are responsible for the stopping the asymptotic freedom.

At first sight, even having this idea would not help much since very little is known on the precise nature of these nonperturbative fields. In particular, the instanton calculus was known to be very sensitive to details of an infrared cut off¹³. The way out of this difficulty was not to try to calculate the nonperturbative fields but describe them instead phenomenologically in terms of a few parameters.

One can understand the trick by considering the same Feynman graphs for the $\Pi_j(Q^2)$. Namely, turn to the graph with a gluon exchange. To ensure that it is determined by short distances we assumed that the momentum brought by the current is large and space-like, $Q^2 \gg \Lambda_{QCD}^2$. Let us consider now in more detail, how this momentum is transported along the quark and gluon lines. The typical case, favored by the phase space, is when all momenta are of order Q . However, there is also a possibility that the large momentum is carried by the quark lines alone while the gluon is soft, $k^2 \ll Q^2$. Then all the points along the quark lines are actually very close to each other in space-time, $\Delta x \sim 1/Q$, while the gluon line travels far away on this scale.

Thus, under the circumstances the graph can be depicted as in Fig. 4. Moreover, there is no reason any longer to use the perturbative expression for the

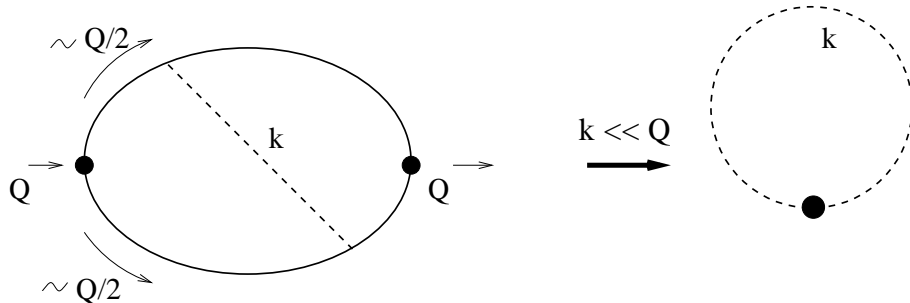


Figure 4: Space-time picture corresponding to the Wilson operator product expansion. The gluon, which is represented by a dashed line, is much softer than the quarks.

gluon line since it is soft and modified strongly by the confinement. The quark lines propagating short distances become a receiver of long wave gluon fields in the vacuum. The receiver is well understood because of the asymptotic freedom. The intensity of the gluon fields is measured. It is characterized by the so called gluon condensate:

$$\langle 0 | \alpha_s \left((\vec{H}^a)^2 - (\vec{E}^a)^2 \right) | 0 \rangle \neq 0 \quad (19)$$

where \vec{H}^a, \vec{E}^a are color magnetic and electric fields.

What is described here in words, has an adequate formulation in terms of the Wilson operator expansion¹⁴, which allows for a systematic and straightforward calculation of the contribution of the gluon condensate (19) to various correlation functions. With account of the gluon condensate the sum rules become:

$$\int R_j(s) \exp(-s/M^2) ds \approx (Parton\ model)$$

$$\times \left(1 + a_j \frac{\alpha_s(M^2)}{\pi} + c_j \frac{\langle 0 | \alpha_s (G_{\mu\nu}^a)^2 | 0 \rangle}{M^4} + \dots \right) \quad (20)$$

where the coefficients a_j, c_j depend on the channel and the ellipses stand for higher order perturbative corrections as well as for power corrections of higher order in M^{-2} . Eqs. (20) are the QCD sum rules². The first check was to see whether the gluon condensate explains the difference in duality intervals in terms of s in the ρ - and J/ψ - channels (see above). It did explain this difference immediately and since that moment on we did not doubt that the gluon condensate is a real thing. It is worth mentioning that in the world of hadrons made of light quarks, the quark condensate, $\langle 0 | \bar{q}q | 0 \rangle$, is the same important as the gluon condensate². But, essentially, this is the only parameter which should be added.

The QCD sum rules turned to be a very straightforward and successful tool for orientation in the hadronic world (for further references see, e.g.,¹⁵). In a simple construct, there were unified the perturbative physics of short distances encoded now in the coefficients c_j and the physics of the soft fields encoded in the quark and gluon condensates. On the theoretical side, the road was open to introduce and treat consistently power corrections via the Wilson operator product expansion (earlier, the studies of the OPE¹⁶ had been confined to the perturbation theory).

Generalizations

We are jumping now over more than 15 years and discuss very briefly a remarkably simple technique which allows to consider power corrections to various observables directly in Minkowski space. The technique is based on introduction of a (fictitious) gluon mass $\lambda, \lambda \rightarrow 0$ and tracing terms non-analytical in λ^2 . The idea is similar in fact to that underlying introduction of the gluon condensate.

Indeed, let us consider again the gluon condensate but this time in one-loop approximation and in case of a finite gluon mass. Obviously enough, it diverges wildly in the ultraviolet. Let us define the gluon condensate, however, as the non-analytical in λ^2 part of the perturbative answer¹⁷:

$$\langle 0 | \alpha_s (G_{\mu\nu}^a)^2 | 0 \rangle = \frac{3\alpha_s}{\pi^2} \int_0^\infty \frac{k^4 dk^2}{(k^2 + \lambda^2)} \equiv -\frac{3\alpha_s}{\pi^2} \lambda^4 \ln \lambda^2, \quad (21)$$

Moreover, to finally get rid of the gluon mass (which is not a pleasant sight for a theorist's eye) we make a replacement:

$$\alpha_s \lambda^4 \ln \lambda^2 \rightarrow c_4 \Lambda_{QCD}^4 \quad (22)$$

where c_4 is an unknown coefficient^b. The central point is that if we evaluate various correlation functions (8) and isolate the terms $\lambda^4 \ln \lambda^2$ we would reproduce the sum rules (20). Indeed, as we explained above the gluon condensate in the sum rules (20) parameterizes the infrared sensitive part of the Feynman graph

^b The technique with $\lambda \neq 0$ is in fact close to the infrared renormalons¹⁸. In the context of the gluon condensate, the infrared renormalons were considered first in Ref.¹⁹.

associated with a soft gluon line. Picking up terms non-analytical in λ^2 is the same good for this purpose since the non-analyticity in λ^2 can obviously arise only from soft gluons, $k \sim \lambda$.

So far we have not got any new result, though. However, the advantage of introducing $\lambda \neq 0$ is that calculations can be performed now in Minkowski space as well and apply for this reason to a much wider class of observables than the OPE underlying the QCD sum rules (20)^c. The link to QCD is again through the bald replacement of the non-analytical in λ^2 terms by corresponding powers of Λ_{QCD} :

$$\begin{aligned} \alpha_s \sqrt{\lambda^2} &\rightarrow c_1 \Lambda_{QCD} \\ \alpha_s \lambda^2 \ln \lambda^2 &\rightarrow c_2 \Lambda_{QCD}^2 \dots \end{aligned} \quad (23)$$

where $c_{1,2}$ are some coefficients treated as phenomenological parameters. The gluon condensate appears now merely as one of the terms in this sequence.

The phenomenology based on such rules turns successful (see, e.g.,²¹ and references therein). The problem is not so much a lack of success but rather too much of overlap²² with old-fashioned hadronization models, like the tube model.

The success of this, most naive approach reveals that at least in the cases when the technique applies the nonperturbative effects reduce to a simple amplification of infrared sensitive contributions to the Feynman graphs.

Limitations

This simple picture is not universally true, however. We knew it since the same year 1979 when the QCD sum rules were formulated. The original papers on the sum rules were 300 typewritten pages long. And many times we were asked, how could we write such a long treatise. It might have been not simple indeed (keeping in mind, for example, that we made typewriting ourselves and only during weekends when we could find a free typewriter). However, the right question would be, I think, why we did not write the papers much longer. Indeed, if you overstep a reasonable length of, say, 30 pages and go after 300 pages, then a curious person should have asked, it seems, why the papers are not, say, 3000 pages long. In fact, there was a well defined answer to this never-asked question: we found a first channel where the sum rules seemed not to work. By some irony, we failed to explain appearance of a large mass scale related to the penguins graphs, see¹. Thus we decided to make a pause to write up the cheerful part of the story and contemplate longer about the emerging difficulties.

The difficulties did not disappear, however, by themselves but rather deepened when we, together with Victor Novikov, came back to the problem. Certainly, there exist channels where the infrared sensitive corrections described above cannot be the whole story. For example, we were able to show²³ that the leading

^cThere is a price to pay, however. Infrared renormalons if applied directly in the Minkowski space do not allow for model independent relations between power corrections to different observables. The reason is that the coupling α_s refers now to infrared region and is, therefore, of order unit. As a result, all orders in α_s are equally important, see, e.g.,²⁰.

power correction in the 0^+ -gluonium channel is given by:

$$\begin{aligned} & \int ImG(s) \exp(-s/M^2) ds/s \approx \\ & \approx G(M^2)_{\text{parton model}} \left[1 + \frac{\langle 0 | \alpha_s (G_{\mu\nu}^a)^2 | 0 \rangle}{M^4} \left(-\frac{2\pi^2}{\alpha_s(M^2)} + \frac{16\pi^2}{b_0 \alpha_s^2(M^2)} \right) \right]. \end{aligned} \quad (24)$$

where

$$G(Q^2) \equiv i \int d^4x \exp(iqx) \langle 0 | T \{ (G_{\mu\nu}^a(x))^2, (G_{\mu\nu}^a(0))^2 \} | 0 \rangle. \quad (25)$$

The power correction proportional to the gluon condensate originates here from two sources. First, there is a standard OPE correction (see (20)) and, second, the one evaluated via a low energy theorem specific for this particular channel. The correction which is not caught by the standard OPE is about (20-30) times larger!

Thus, if we characterize the scale where the asymptotic freedom gets violated by the power correction by the value M_{crit}^2 where these corrections become, say, 10%, then M_{crit}^2 differs drastically in various channels:

$$(M_{crit}^2)_{\rho\text{-meson}} \approx 0.6 \text{ GeV}^2, \quad (M_{crit}^2)_{0^+ \text{ gluonium}} \approx 15 \text{ GeV}^2. \quad (26)$$

Thus, the proof of the low-energy theorem brought a proof of existence of qualitatively different scales in the hadron physics²³.

What made the search for the “exceptional” channels where the OPE fails to identify the leading correction so difficult was lack of any systematic way to evaluate the power corrections beyond the same OPE. For example, the huge power correction in (24) looked a stranger since a single power correction cannot match a resonance but we were unaware of the source of (hypothetical) other corrections of the same mass scale. Nevertheless, through a meticulous analysis²³ we were able to find hints that there exists a hierarchy of the strengths of the extra power corrections in the “exceptional” channels. Moreover, this hierarchy could be explained in terms of the transitions of the corresponding currents directly to instantons, Fig. 5 (see also²⁴). Although qualitatively the new extended picture worked well we left the field disappointed by the lack of a machinery to evaluate the new effects. Later, a model of instanton liquid was developed that allowed for a much more quantitative treatment of the instanton effects (for a review and further references see²⁵). The model appears to be successful phenomenologically.

Elusive Effects of Confinement. Short Strings?

Looking backward, it still remains a mystery whether any specific confinement effects are revealed through the power corrections. Indeed, consider the vacuum of pure gluodynamics. It is known from lattice measurements that external heavy quarks are confined by this medium (for a review see, e.g.,²⁶).

On the other hand, the effects included into the sum rules so far do not seem to encode the confinement. Indeed, the perturbative QCD resembles ordinary bremsstrahlung in QED. The gluon condensate, as well as other newly found

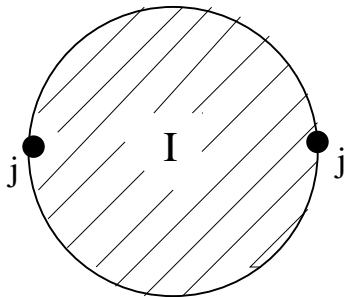


Figure 5: Direct instantons. One substitutes instanton fields, both bosonic and fermionic into the currents and integrates over the instanton sizes.

power corrections can be detected by introducing a fictitious gluon mass which is not sensitive to the non Abelian nature of gluons at all. Finally, instantons are known not to ensure the confinement either ²⁷.

In an attempt to find power corrections related more directly to the physics of confinement one can turn ²⁸ to the Abelian Higgs model (AHM) which underlies the dual superconductor model of the confinement ⁷. If one introduces a pair of external magnetic charges into the vacuum of the AHM model in the Higgs phase then the potential grows at large distances, see Eq. (3). The scale of distances is set up by the inverse masses of the vector and scalar fields, $m_{V,S}^{-1}$. This growth of the potential is due to the Abrikosov-Nielsen-Olesen strings ⁸. The relevance to non-Abelian gauge theories is most transparent in the so called $U(1)$ projection which treats the diagonal gluon fields like photons ⁵. There is ample evidence in favor of this picture on the lattice ³¹.

Consider now *short* distances, $r \ll m_{V,S}^{-1}$. Then the Coulomb like interaction dominates. However, there is a stringy correction to the potential at any small distances ²⁸:

$$\lim_{r \rightarrow 0} V(r) = -\frac{Q_M^2}{4\pi r} + \sigma_0 r \quad (27)$$

where Q_M is the magnetic charge. The ANO string is a bulky object on this scale and is not responsible for the linear correction. Instead, the stringy potential at short distances is due to infinitely thin topological strings which connect the magnetic charges and which are defined through vanishing of the scalar field along the string. Thus, it turns out that at least in this model the confined charges learn about confinement already at small distances because of the short strings which are seeds for future confining ANO strings. Amusingly enough, it demonstrates that at short distances a dimension two quantity is not necessarily the gluon mass squared but could be a string tension as well. The notion of such a string might be generalized to the QCD ²⁸.

The linear potential at short distances (27) corresponds in the momentum space to a $1/Q^2$ correction ³⁰. Moreover, it can be imitated by a *short-distance* gluon mass. To reproduce the positive string tension σ_0 at short distances, it

should be however a tachyonic mass³²! This, openly heuristic assumption allows to extend the phenomenology of the $1/Q^2$ corrections. In particular, there arises a $1/M^2$ term missing from the standard sum rules (20):

$$\int R_j(s) \exp(-s/M^2) ds \approx (Parton\ model) \times \left(1 + a_j \frac{\alpha_s(M^2)}{\pi} + \frac{b_j}{M^2} + c_j \frac{\langle 0 | \alpha_s (G_{\mu\nu}^a)^2 | 0 \rangle}{M^4} + \dots \right) \quad (28)$$

where the coefficients b_j are now calculable in terms of the tachyonic gluon mass. The modified sum rules turn to be successful phenomenologically provided that $\lambda^2 \approx -0.4 GeV^2$ ³². In particular, in the O^+ -gluonium channel a new correction arises which matches the large correction in Eq. (24) which so far was hanging without any support from other known power-like terms. In some channels, the new terms may compete with the direct instanton contributions. Further checks are necessary, however, before one can be certain about the existence of the novel $1/Q^2$ corrections associated with short distances^d.

Conclusions.

The QCD sum rules even now seem to provide a standard framework to

- (i) Analyze infrared sensitive power corrections to various correlation functions and get oriented in properties of hadrons with various quantum numbers,
- (ii) To look for further contributions which go beyond the quark and gluon condensates.

The nature may turn to be generous as far as power corrections are concerned. I am borrowing this term from a talk on dark matter. Indeed, first people assumed that there should be a single dominant source of the dark matter, and now it appears distributed among various equally important components. Similar picture may be true for the sum rules. Indeed, very first idea would be that theoretically the correlation functions $\Pi_j(Q^2)$ could be found perturbatively at large Q^2 and the growth of the effective coupling at smaller Q^2 would signal the breaking of the asymptotic freedom. Then the picture got more involved and the effect of soft non-perturbative fields was included in terms of the quark and gluon condensates. It appears not suffice to explain the peculiarities of all the channels and the effect of direct instantons was invoked. As the latest development, hypothetical $1/Q^2$ corrections associated with short strings are established within the Abelian Higgs model which is thought to mimic the QCD confinement.

Thus it appears now that all three “ingredients” of QCD mentioned in the introduction have already found their way into the sum rules. And each time there are claims of some qualitative effects getting explained. It might be not the end of the story.

^dUnconventional $1/Q^2$ corrections were introduced also within other frameworks, such as ultra-violet renormalons, Nambu-Jona-Lasinio model, or modified effective coupling, see, e.g., Ref. ³³ and references therein. The tachyonic gluon mass can be considered as a particular prescription to fix such corrections in terms of a single parameter.

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